Student Pages

List of Problems

List of Problems (cont.)

Types of Triangles

Task: To develop definitions for right, acute, obtuse, isosceles, and equilateral triangles.

Procedure:

- Construct a random right triangle.
- Make measurements on the triangle.
- Record your drawing of the triangle and its measurements in the space provided.
- Repeat this process with other right triangles.
- Based on your findings, develop a conjecture about right triangles and write it below.
- Repeat this procedure for acute, obtuse, isosceles, and equilateral triangles.

Drawings & Data

Conjectures

A right triangle is a triangle which... _____

_____

An acute triangle is a triangle which... _____

_____

An obtuse triangle is a triangle which... _____

_____

An isosceles triangle is a triangle which... _____

_____

An equilateral triangle is a triangle which... _____

_____

T 2
Angle Measurements

Task: To explore the relationship among the interior angles in different types of triangles.

Procedure:

- Construct a triangle.
- Measure each angle.
- Draw the triangles and record the angle measurements on the chart below.
- Repeat this procedure on five other triangles.
- On the following page, state conjectures about your findings.

Triangle Drawings	∠ABC	∠BCA	∠CAB
1.			
2.			
3.			
4.			
5.			
6.			

Conjectures

State a conjecture about the sum of the three angles.

State other conjectures about the three angles or the relationship between the sum of two angles and the third angle.

T 3
Interior and Exterior Angles

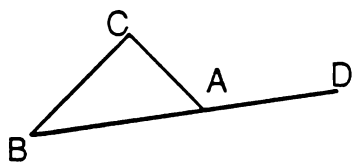
Task: To explore the relationship among the three interior angles and one exterior angle in different types of triangles.

Procedure:

- Construct an acute ΔABC.
- Draw an extension of side $\overline{BA}$ such that BA = AD.
- Measure the angles and record the measurements in the chart below.
- Repeat the steps for other types of triangles. Use the Repeat option.
- For each triangle, measure the interior angles of ΔABC and the exterior $\angle CAD$.
- State your conjectures about the relationship between the exterior and interior angles.

Diagram:

On an acute triangle, $\overline{BA}$ is extended from A such that BA = AD. *$\angle CAD$ created by the extension is called an exterior angle.*



Triangles	exterior $\angle CAD$	$\angle ABC$	$\angle BCA$	$\angle CAB$
Right				
Acute				
Obtuse				
Isosceles				
Equilateral				

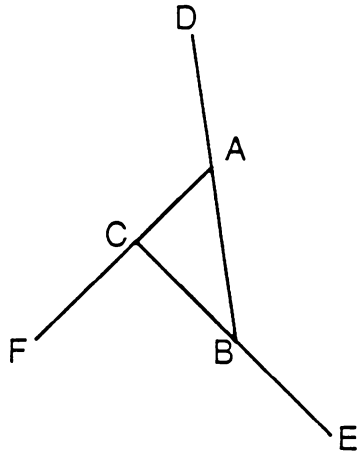
Conjectures

Exterior Angles

Task: To investigate the relationship among the three exterior angles of triangles.

Procedure:

- Construct an acute triangle.
- Draw the three exterior angles.
- Measure the exterior angles.
- Record drawings and measurements.
- Repeat the steps for other types of triangles.
- State your conjectures.



Drawings & Data

Conjectures

What is the sum of the measures of the three exterior angles for an acute triangle? _____

Is this sum the same for all types of triangles? _____

Triangles Whose Sides Are Three Consecutive Integers

Task: To discover whether sides whose lengths are three consecutive integers form triangles. When they do, to investigate the types of triangles they form.

Procedure:

- Construct a triangle using the side-side-side option.
- Use side lengths that are consecutive integers (*e.g.*, 4-5-6).
- Record your drawing of the triangle and the lengths of the sides.
- Repeat the steps for several other sets of consecutive integers.
- State your conjectures.

Drawings & Data

Conjectures

Do three consecutive integers always create a triangle? Why?

Do consecutive integers that do create triangles, create the same type of triangle? Why?

Scalene Triangles

Task: To make conjectures about the relative sizes of angles in scalene triangles given the lengths of the sides. (A *scalene triangle* has no equal sides.)

Procedure:

- Make initial conjectures:
  In ΔABC, side $\overline{AB}$ is the shortest, side $\overline{BC}$ is the middle, and side $\overline{AC}$ is the longest.
  Which angle will be the smallest?
  Which angle will be the largest?
- Construct a triangle using the side-side-side option.
- Construct a triangle with no equal sides.
- Measure the angles if you need to.
- Were your predictions accurate?
- State your conjectures.

Drawings & Data

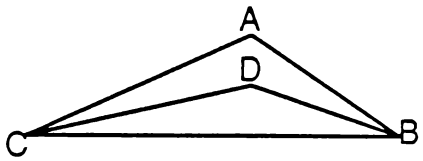
Conjectures

Angles in Triangles

Task: To explore the measure of a vertex angle, created from a random point inside a triangle.

Procedure:

- Construct a ΔABC.
- Label a random point D inside ΔABC and draw $\overline{BD}$ and $\overline{CD}$.
- Measure $\angle BDC$ and $\angle BAC$.
- Record the drawing and angle measurements.
- Repeat this procedure (do NOT use the Repeat option) on four other triangles.
- State your conjectures.



Triangle Drawings	$\angle BDC$	$\angle BAC$
1.		
2.		
3.		
4.		
5.		

Conjectures

Angles Formed by Subdividing Triangles

Task: To explore angles formed by subdividing a triangle.

Procedure:

- Construct a ΔABC. Measure $\angle BAC$.
- Place a random point D on $\overline{BC}$ of ΔABC.
- Draw $\overline{AD}$. Subdivide $\overline{AD}$ into four parts, using the Label option.
- Measure and draw $\angle BEC$, $\angle BFC$, and $\angle BGC$.
- Record the drawing and the measurements of the angles.
- Repeat the procedure with other triangles.
- State your conjectures.

Triangle Drawings	$\angle BAC$	$\angle BEC$	$\angle BFC$	$\angle BGC$
1.				
2.				
3.				
4.				
5.				

Conjectures

State a conjecture about the relationship between $\angle BAC$ and the other angles.

_____

_____

_____

_____

Angle-Side-Angle

Task: To investigate the measurements that produce triangles using angle-side-angle.

Procedure:

- Use the measurements listed in the chart below.
- Construct a triangle using the angle-side-angle option.
- Record the drawing of each triangle on the chart.
- State your conjectures on the following page.

Triangle Drawings	∠BAC	AB	∠CBA
1.	100	1	45
2.	100	1	55
3.	100	1	65
4.	100	0.5	75
5.	80	3	40
6.	80	3	60
7.	80	3	80
8.	80	3	100
9.			

Conjectures

In order to make a triangle, are there any constraints or limits that should be placed on the measures of $\angle BAC$, $\angle CBA$, or $\overline{AB}$?

Side-Angle-Side

Task: To investigate the measurements that produce triangles using side-angle-side.

Procedure:

- Use the measurements listed in the chart below.
- Construct a triangle using the side-angle-side option.
- Record the drawing of each triangle on the chart.
- State your conjectures on the following page.

Triangle Drawings	AB	∠BAC	AC
1.	7	20	4
2.	7	40	4
3.	7	60	4
4.	7	90	4
5.	7	110	4
6.	5	100	5
7.	5	90	5
8.	5	80	5
9.	5	20	5

Conjectures

In order to make a triangle, are there any constraints or limits that should be placed on the measures $\overline{AB}$, $\angle BAC$, or $\overline{AC}$?

Sides of Right Triangles

Task: To observe patterns among the lengths of the sides in a right triangle.

Procedure:

- Construct a right ΔABC.
- Measure the lengths of the sides.
- Record your drawings and measurements in the table below.
- Repeat on four other right triangles.
- State your conjectures.

Triangle Drawings	AB	AC	BC
1.			
2.			
3.			
4.			
5.			

Conjectures

State conjectures about the relationships between the length of the side opposite the right angle and the lengths of the other two sides.

Side-Side-Side

Task: To investigate the measurements that produce triangles using side-side-side.

Procedure:

- Construct a ΔABC using the side-side-side option.
- If a triangle is formed, state what type it is.
- Record the lengths and the squares of the lengths in the table below.
- Repeat for three other triangles.
- State your conjectures.

Triangle Drawing	Lengths AB AC BC	Squares of Lengths $(AB)^2$ $(AC)^2$ $(BC)^2$	Forms a Triangle?	Type of Triangle

Conjectures

State conjectures about which combinations make triangles, which do not, and what type of triangle is formed.

Supporting Conjectures

Task: To collect data to support conjectures.
To evaluate conditions under which a statement is true.

Procedure:

- The conjectures below are **always true**(A), **sometimes true**(S), or **never true**(N).
- Read each conjecture and place the appropriate letter (A, S, or N) next to each statement.
- Justify your answer using one of the following procedures:
  If you think a conjecture is **always true** (A), provide three examples.
  If you think a conjecture is **sometimes true** (S), provide two examples (one true and one false).
  If you think that a conjecture is **never true** (N), provide a counter-example.
- Be sure that your examples contain the appropriate measurements.

Conjectures

_____ 1. A scalene triangle is not a right triangle.

_____ 2. An isosceles triangle is not a right triangle.

_____ 3. An exterior angle of a triangle has a measure greater than the measure of any interior angle.

_____ 4. An exterior angle of a triangle has a measure greater than the measure of any remote interior angle.

_____ 5. The longest side of a right triangle is the side opposite the right angle.

_____ 6. In any triangle, a median drawn from a vertex to a side bisects that side.

_____ 7. In any triangle, a median drawn from a vertex to a side bisects the angle at the vertex.

T 14
Area of a Triangle

Task: To understand this formula for computing the area of a triangle:

Area = $\frac{1}{2}$ x (measure of the base) x (measure of the altitude) or $A = \frac{bh}{2}$

(**Definition:** Any side of a triangle can be called a *base*. Given a base, a line segment drawn from the remaining vertex, perpendicular to the base or the extension of the base, is called an *altitude* (or *height*) of the triangle.)

Procedure:

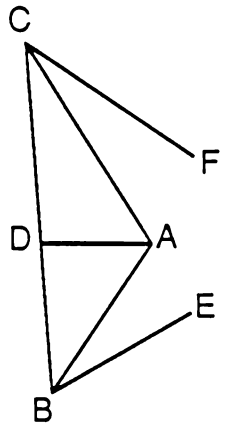
- Construct an isosceles ΔABC. Draw an altitude from vertex A.
- Measure the altitude and the base.
- Calculate the value of bh/2, using a calculator.
- Record your data in the chart.
- Repeat this procedure by drawing the altitude from B. Then do the same for vertex C.
- Check your calculations using the Area option to measure the area.
- Repeat this procedure for an acute triangle. Then repeat for an obtuse triangle.
- State your conjectures on the following page.

Triangle Drawings	Base	Altitude	Area (by formula)	Area (w/Area option)
Isosceles Triangle				
Acute Triangle				
Obtuse Triangle				

Conjectures

Altitudes of Obtuse Triangles

Task: To explore the construction formed by altitudes of obtuse triangles.



Procedure:

- Construct obtuse ΔABC.
- Draw altitudes $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$.
- Look for angles that might be congruent.
- Measure the angles.
- Record your drawings and data below.
- State your conjectures.

Drawings & Data

Conjectures

State a conjecture about congruent angles in this figure (other than the right angles).

Triangles from Altitudes

Task: To investigate the triangle formed by joining the endpoints of the three altitudes.

Procedure:

- Construct an acute ΔABC.
- Draw an altitude from each vertex.
- Erase the altitudes, leaving the labels D, E, and F.
- Now draw ΔDEF.
- Determine whether the ΔDEF extends beyond the original ΔABC.
- Check whether ΔDEF is a right triangle.
- Repeat on several other triangles.

Drawings & Data

Conjectures

In what kind of triangle does the region defined by ΔDEF extend beyond the exterior of ΔABC?

_____

_____

Can ΔDEF ever be a right triangle? _____

Suppose ΔABC is an isosceles triangle with vertex $\angle A$. When the vertex angle of ΔABC is close to 90°, the perimeter of ΔDEF will be close to what measure? _____

Angle Bisectors

Task: To understand an angle bisector as a set of points that share a certain property.

Procedure:

- Construct a right ΔABC.
- Draw angle bisector $\overline{AD}$.
- Place a random point E on the bisector.
- Measure the distance from E to $\overline{AB}$ and the distance from E to $\overline{AC}$.
- Record your data and the drawing.
- Repeat for each triangle listed. Do NOT use the Repeat option.
- State your conjectures on the next page.

Triangle Drawings		Distance from E to $\overline{AB}$	Distance from E to $\overline{AC}$
Right			
Acute			
Obtuse			
Isosceles			
Equilateral			
Scalene			

Conjectures

Medians

Task: To understand a median as a set of points that share a certain property.

Procedure:

- Draw a ΔABC.
- Draw median $\overline{AD}$. Label a random point E on median $\overline{AD}$.
- Draw $\overline{CE}$ and $\overline{BE}$.
- Measure the areas of the triangles inside ΔABC.
- Record your data.
- State your conjectures.

Drawings & Data

Conjectures

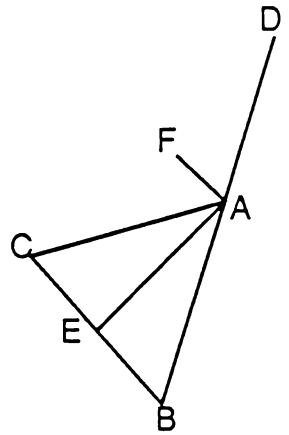
State conjectures about the relationships among the areas of the triangles inside ΔABC.

Angle Bisectors at a Vertex

Task: To explore the angle formed by the angle bisector of the exterior and interior angles at a vertex.

Procedure:

- Construct acute ΔABC.
- Extend $\overline{AB}$ from A.
- Draw $\overline{AE}$, the bisector of $\angle BAC$.
- Draw $\overline{AF}$, the bisector of $\angle CAD$.
- Measure $\angle FAE$.
- State your conjectures.



Drawings & Data

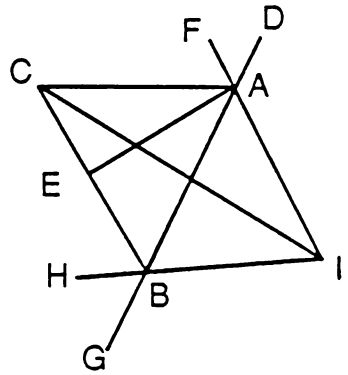
Conjectures

Angle Bisectors of Exterior Angles

Task: To explore properties of the intersection point of the angle bisectors of two exterior angles.

Procedure:

- Construct acute ΔABC.
- Extend $\overline{AB}$ from A.
- Draw $\overline{AE}$, the bisector of $\angle BAC$.
- Draw $\overline{AF}$, the bisector of $\angle CAD$.
- Extend $\overline{AB}$ from B.
- Draw $\overline{BH}$, the bisector of $\angle GBC$.
- Label the intersection of the two bisectors ($\overline{FA}$ and $\overline{HB}$) of the exterior angles for ΔABC with point I.
- Draw $\overline{IC}$.
- State conjectures related to segment $\overline{IC}$.



Drawings & Data

Conjectures

Angle Bisectors and Side Lengths

Task: To explore the way an angle bisector divides the opposite side.

Procedure:

- Use the measurements from the chart below.
- Construct a ΔABC using the side-angle-side option.
- Draw $\overline{AD}$, the bisector of $\angle BAC$.
- Observe the relative lengths of the segments $\overline{BD}$, $\overline{CD}$, $\overline{AB}$, and $\overline{AC}$.
- Record the drawings.
- State your conjectures.

Triangle Drawings	AB	∠BAC	AC
1.	8	100	2
2.	8	100	3
3.	8	100	4
4.	8	100	5
5.	8	100	6
6.	8	100	8

Conjectures

State conjectures about the relationships among $\overline{BD}$, $\overline{CD}$, $\overline{AB}$, and $\overline{AC}$, relative to the location of point D on $\overline{CB}$.

_____

_____

_____

Triangles Formed by an Angle Bisector

Task: To explore the relationships between the two triangles formed when an angle bisector is drawn in a triangle.

Procedure:

- Construct an isosceles ΔABC.
- Draw $\overline{AD}$ bisecting $\angle CAB$.
- Measure the lengths, areas, and perimeters.
- Record your data.
- Repeat this procedure on other types of triangles.
- State your conjectures.

Triangle Type	LENGTHS AB	AC	BD	DC	Area ΔABD	Area ΔACD	Perimeter ΔABD	Perimeter ΔACD
Isosceles								
Acute								
Right								
Equilateral								
Obtuse								

Conjectures

T 23
Angle Bisectors in Scalene Triangles

Task: To discover proportions among the elements and triangles in a scalene triangle with one angle bisector.

Procedure:

- Construct a scalene ΔABC.
- Draw the angle bisector of angle A.
- Measure the segments and triangles formed.
- Record your data.
- State your conjectures about relationships among the elements of the figure.

Drawings & Data

Conjectures

The Triangles Created by Medians

Task: To explore the relationships among the elements (sides, segments, angles) and the properties (area, perimeter) of the three triangles (ΔABC, ΔABD, ΔCAD) created when a median is drawn in a triangle.

Procedure:

- Construct a right ΔABC.
- Draw the median $\overline{AD}$.
- Measure elements, area, and perimeter.
- Repeat on other triangles.
- State your conjectures on the following page.

Drawings & Data

The Triangles Created by Medians

Conjectures

Right Triangles (non-isosceles):

_____

_____

Acute Triangles:

_____

_____

Obtuse Triangles:

_____

_____

Isosceles Triangles:

_____

_____

In an isosceles triangle, the median from the vertex angle can also be a _____ ,
a_____ , or a _____ .

Equilateral Triangles:

_____

_____

In an equilateral triangle, the median from any vertex of the triangle can also be a _____ ,
a _____ , or a _____ .

T 25
Two Medians

Task: To explore relationships in a triangle when two medians are drawn.

Procedure:

- Construct ΔABC.
- Draw medians $\overline{AD}$ and $\overline{BE}$.
- Label the intersection of the two medians with point F.
- State your conjectures.

Drawings & Data

Conjectures

T 26
A Median and Sides

Task: To investigate the relationship between the length of the median and the lengths of the sides.

Procedure:

- Construct any ΔABC.
- Draw median $\overline{AD}$.
- Measure the length of the median and the lengths of the sides of the triangle.
- State your conjectures about these measurements.
- Investigate whether your conjectures hold true for different types of triangles.

Drawings & Data

Conjectures

Three Medians

Task: To investigate the relative lengths of medians in a triangle.

Procedure:

- Construct any ΔABC.
- Draw the three medians.
- Measure the lengths of the medians and the lengths of the sides of the triangle.
- State your conjectures.

Drawings & Data

Conjectures

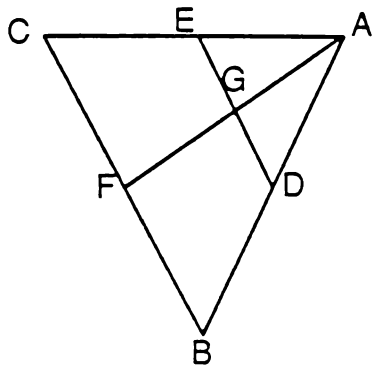
In relation to the lengths of the sides, which median do you think will be the shortest? Which median do you think will be the longest?

T 28
One Midsegment

Task: To explore figures formed by drawing one midsegment in a triangle.

Procedure:

- Construct any ΔABC.
- Draw midsegment $\overline{DE}$ connecting $\overline{AB}$ and $\overline{AC}$.
- Label the midpoint of $\overline{BC}$ with point F.
- Draw $\overline{AF}$ and label the intersection of $\overline{DE}$ and $\overline{AF}$ with point G.
- Measure the elements of the figure.
- Record your data.
- State your conjectures.



Drawings & Data

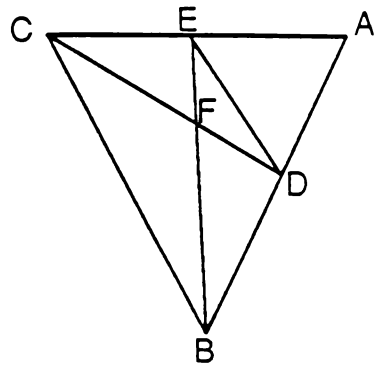
Conjectures

One Midsegment and Two Medians

Task: To explore figures formed by a midsegment and two medians.

Procedure:

- Construct any ΔABC.
- Draw midsegment $\overline{DE}$ connecting $\overline{AB}$ and $\overline{AC}$.
- Draw $\overline{BE}$ and $\overline{CD}$.
- Label the intersection of $\overline{BE}$ and $\overline{CD}$ with point F.
- Measure the elements of the figure.
- Record your data.
- State your conjectures.



Drawings & Data

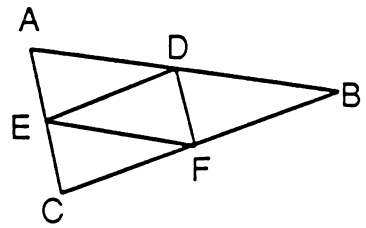
Conjectures

T 30
Three Midsegments

Task: To explore figures formed by a triangle with the three midsegments.

Procedure:

- Construct any ΔABC.
- Draw midsegment $\overline{DE}$ connecting $\overline{AB}$ and $\overline{AC}$.
- Subdivide $\overline{BC}$ into two equal parts.
- Draw $\overline{DF}$ and $\overline{EF}$.
- Measure the resulting segments and triangles.
- Record your data.
- State your conjectures.



Drawings & Data

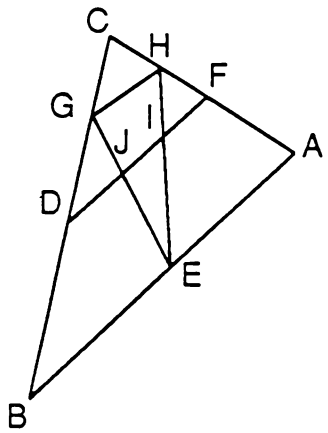
Conjectures

Midsegments and Midpoints

Task: To explore figures formed by a triangle and the midpoint of each side.

Procedure:

- Construct an acute ΔABC.
- Label the midpoints of $\overline{BC}$, $\overline{AB}$, and $\overline{AC}$ with points D, E, and F respectively.
- Draw $\overline{DF}$.
- Label the midpoints of $\overline{CD}$ and $\overline{CF}$ with points G and H respectively.
- Draw $\overline{GH}$, $\overline{EG}$, and $\overline{EH}$.
- Label the intersection of $\overline{EH}$ and $\overline{DF}$ with point I.
- Label the intersection of $\overline{EG}$ and $\overline{DF}$ with point J.
- Measure the resulting segments and triangles.
- Record your data.
- State your conjectures.

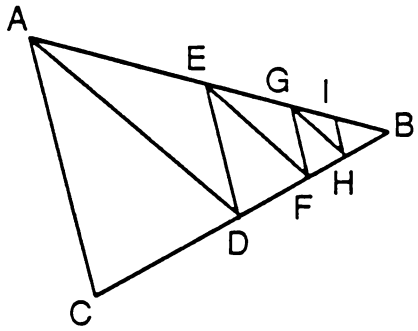


Drawings & Data

Conjectures

Parallel Lines

Task: To reproduce the figure below, **without** using the Parallel option. In ΔABC, $\overline{DE} \parallel \overline{FG} \parallel \overline{HI}$ and $\overline{AD} \parallel \overline{EF} \parallel \overline{GH}$.



Procedure:

- Make a drawing similar to the figure above.
- Collect data to confirm that the segments are parallel.
- Describe below the procedures for reproducing the figure.
- Develop three methods for drawing parallel lines without using the Parallel option.

Drawings & Data

Conjectures

Procedure for reproducing the figure:

Three methods for drawing parallel lines inside triangles **without** using the Parallel option. Describe your methods and record sample drawings for each method.

1.

2.

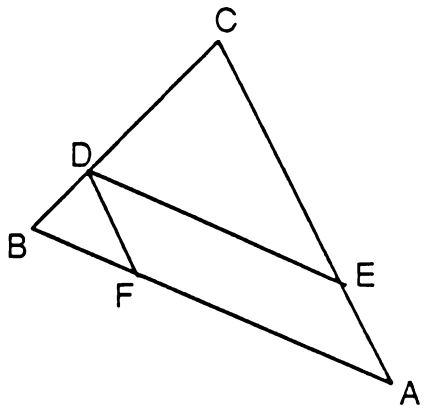
3.

Parallelogram in a Triangle

Task: To investigate parallelograms drawn in isosceles triangles.

Procedure:

- Construct an isosceles ΔABC using the side-angle-side option so that AC = AB.
- Label a random point on $\overline{BC}$.
- Construct $\overline{DE}$ such that $\overline{DE}$ is parallel to $\overline{AB}$.
- Construct $\overline{DF}$ such that $\overline{DF}$ is parallel to $\overline{AC}$.
- Measure the sum of the lengths AB + AC.
- Measure the perimeter of the parallelogram AEDF.
- Record your data.
- Repeat this procedure for different types of triangles, using a new random point on each triangle.
- State your conjectures.



Drawings & Data

Conjectures

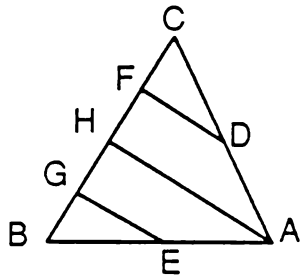
What is the relationship between AB + AC and the perimeter of parallelogram AEDF?

Perpendiculars in a Triangle

Task: To explore the relationship between perpendiculars drawn from midpoints of two sides in a triangle to the third side.

Procedure:

- Construct any ΔABC.
- Label the midpoints of $\overline{AC}$ and $\overline{AB}$ with points D and E respectively.
- Draw $\overline{DF}$ and $\overline{EG}$ so that they are perpendicular to $\overline{BC}$.
- Investigate the relationship between DF and EG.
- Record your data.
- Draw the altitude $\overline{AH}$.
- Investigate the relationships among DF, EG, and altitude AH.
- State your conjectures.

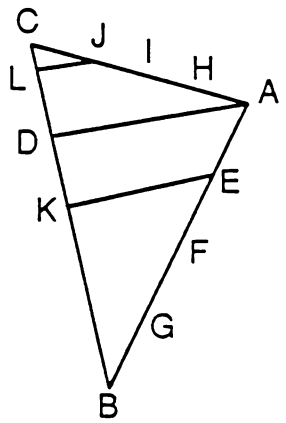


Drawings & Data

Conjectures

Lengths of Perpendiculars - Part A

Task: To investigate relationships among perpendicular line segments drawn from points on one side to another side.



Procedure:

- Study this figure.
- Repeat these constructions on other triangles.
- Measure line segments.
- Record your data.
- State your conjectures.

Drawings & Data

Conjectures

Lengths of Perpendiculars - Part B

Task: To investigate relationships among perpendicular line segments drawn from points on two sides to the third side.

Procedure:

- Construct any ΔABC.
- Subdivide any two sides into three segments.
- Draw perpendiculars from the subdivisions to the third side.
- Repeat this drawing on different types of triangles.
- Repeat the same construction with four subdivisions and five subdivisions.
- Record your data in the chart below.
- State your conjecture.

Number of Subdivisions (n)	Sum of the lengths of the perpendiculars along $\overline{AB}$	Sum of the lengths of the perpendiculars along $\overline{AC}$	Length of Altitude
3			
3			
3			
3			
4			
4			
4			
4			
5			
5			
5			
5			

Conjectures

Perpendicular Bisectors in a Right Triangle

Task: To explore relationships among triangles and elements of triangles formed by the perpendicular bisector of one leg of a right triangle.

Procedure:

- Construct a right triangle.
- Draw the perpendicular bisector of one leg of the triangle.
- Collect data about the elements in the triangles and among the triangles themselves.
- Record your data.
- State your conjectures.

Drawings & Data

Conjectures

Two Perpendicular Bisectors in a Right Triangle

Task: To explore relationships among triangles and elements of triangles formed by the perpendicular bisectors of both legs of a right triangle.

Procedure:

- Construct a right triangle.
- Draw the perpendicular bisector of each leg of the triangle.
- Measure and record data.
- State your conjectures.

Drawings & Data

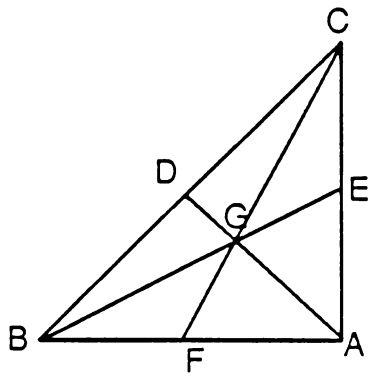
Conjectures

Three Medians in a Right Isosceles Triangle

Task: To explore relationships among triangles and elements of triangles formed by the three medians in a right isosceles triangle.

Procedure:

- Construct a right isosceles ΔABC using the side-angle-side option.
- Draw three medians in the triangle.
- Label the intersection point G.
- Record your data.
- State your conjectures.
- Check your conjectures with another right isosceles triangle.
- Repeat this procedure on an acute triangle.



Drawings & Data

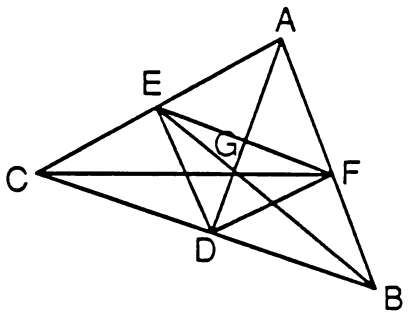
Conjectures

Angle Bisectors in Isosceles Triangles

Task: To explore figures formed by the three angle bisectors in isosceles triangles.

Procedure:

- Construct an isosceles ΔABC.
- Draw the three angle bisectors $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$.
- Draw $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$.
- Label the intersection point of the angle bisectors with point G.
- Measure and record data.
- State your conjectures.



Drawings & Data

Conjectures

Three Altitudes in Acute Triangles - Part A

Task: To explore relationships in figures formed by the three altitudes in acute triangles.

Procedure:

- Construct an acute ΔABC.
- Draw the three altitudes, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$.
- Draw $\overline{EF}$.
- Label the intersection point of the altitudes with point G.
- Measure and record your data about the relationships between the various angles and triangles in the drawing.
- State your conjectures.

Drawings & Data

Conjectures

Three Altitudes in Acute Triangles - Part B

Task: To explore relationships in figures formed by the three altitudes in acute triangles.

Procedure:

- Use your construction from Part A:
  - Construct an acute ΔABC.
  - Draw the three altitudes, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$.
  - Draw $\overline{EF}$.
  - Label the intersection point of the altitudes with point G.
- Draw $\overline{DE}$ and $\overline{DF}$.
- State your conjectures about the relationships between various angles and triangles in the drawing.

Drawings & Data

Conjectures

Three Altitudes in Acute Triangles - Part C

Task: To explore relationships in figures formed by the three altitudes in acute triangles.

Procedure:

- Use your construction from Part B:
  - Construct an acute ΔABC.
  - Draw the three altitudes $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$.
  - Draw $\overline{EF}$.
  - Label the intersection point of the altitudes with point G.
  - Draw $\overline{DE}$ and $\overline{DF}$.
- Circumscribe ΔBFE.
- State your conjectures.

Drawings & Data

Conjectures

How does this circle relate to ΔABC?

Three of Everything

Task: To explore the relationships among the points of intersection of the three **medians**, **altitudes**, **angle bisectors**, and **perpendicular bisectors** in triangles.

Procedure:

- Construct a right triangle.
- Draw the three medians in the triangle.
- Place a check mark under the columns which describe the intersection of the three **medians**.
- Repeat the procedure for other triangles and check the appropriate column on the chart.
- Use the same procedure to complete the chart below for **altitudes**, **angle bisectors**, and **perpendicular bisectors**.
- State your conjectures.

The three **medians** intersect...

Triangle Type	In One Point	Outside Triangle	Inside Triangle	On Triangle
Right				
Acute				
Obtuse				
Isosceles (Acute)				
Isosceles (Obtuse)				
Equilateral				

The three **altitudes** intersect...

Triangle Type	In One Point	Outside Triangle	Inside Triangle	On Triangle
Right				
Acute				
Obtuse				
Isosceles (Acute)				
Isosceles (Obtuse)				
Equilateral				

Three of Everything

The three **angle bisectors** intersect...

Triangle Type	In One Point	Outside Triangle	Inside Triangle	On Triangle
Right				
Acute				
Obtuse				
Isosceles (Acute)				
Isosceles (Obtuse)				
Equilateral				

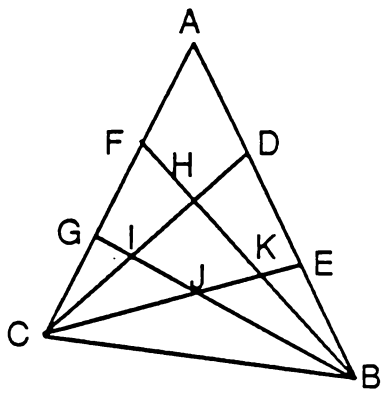
The three **perpendicular bisectors** intersect...

Triangle Type	In One Point	Outside Triangle	Inside Triangle	On Triangle
Right				
Acute				
Obtuse				
Isosceles (Acute)				
Isosceles (Obtuse)				
Equilateral				

Conjectures

Trisecting Sides in a Triangle

Task: To explore figures formed by trisecting two sides of a triangle and connecting these points to the opposite vertex.



Procedure:

- Construct any ΔABC.
- Subdivide any two sides into three equal parts.
- Connect the two vertices to the points of the subdivision.
- Label the four points of intersection formed by these segments to match the drawing.
- Collect and record data.
- State your conjectures.

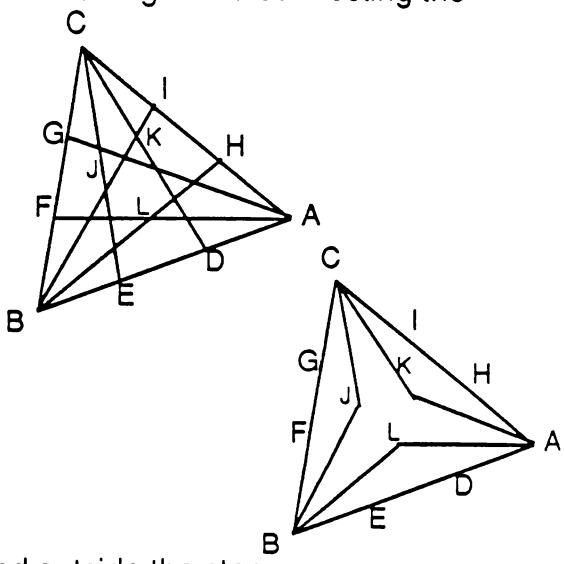
Drawings & Data

Conjectures

State what would happen if you subdivide any two sides into 10 equal parts.

T 43
Star Triangles

Task: To explore figures formed by trisecting the three sides of a triangle and connecting the points to the opposite vertex.



Procedure:

- Construct an equilateral ΔABC.
- Subdivide each side into three equal segments.
- Connect the vertices of ΔABC to the corresponding points of the subdivisions.
- Label the intersection of $\overline{CE}$ and $\overline{BI}$ with point J.
- Label the intersection of $\overline{AG}$ and $\overline{CD}$ with point K.
- Label the intersection of $\overline{AF}$ and $\overline{BH}$ with point L.
- These three intersection points together with the vertices of ΔABC (Figure BJCKAL) form a "star".
- State conjectures about the relationships among the triangles inside the star, among the triangles outside the star, and between the triangles inside and outside the star.
- Repeat this procedure for different types of triangles.
- State your conjectures. Which conjectures hold true for which types of triangles?

Drawings & Data

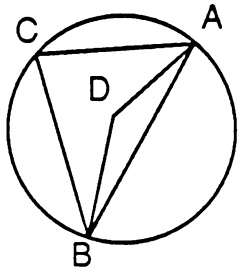
Conjectures

Circumscribed Triangles - Part A

Task: To explore the relationship between a central angle and angles on an arc.

Procedure:

- Construct an acute ΔABC.
- Draw a circumscribed circle with center D.
- Draw $\overline{AD}$ and $\overline{BD}$.
- Record your data.
- State as many conjectures as you can about the relationship of $\angle BDA$ to other angles in the drawing.



Drawings & Data

Conjectures

Circumscribed Triangles - Part B

Task: To explore the relationship between a central angle and angles on an arc.

Procedure:

- Use your construction from Part A.
  - Construct an acute ΔABC.
  - Draw a circumscribed circle with center D.
  - Draw $\overline{AD}$ and $\overline{BD}$.
- Use the Adjustable element option on the Measure menu to locate points E and F on arc ABC. Locate E and F so that DE = DF = AD.
- Record your data.
- State your conjectures.
- Which conjectures are identical to those found in Part A?

Drawings & Data

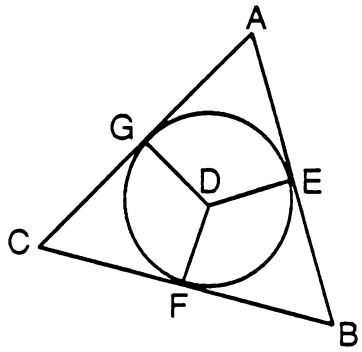
Conjectures

A Triangle and Its Inscribed Circle

Task: To explore the relationship between the center of an inscribed circle and the three tangent points to the triangle in which it is inscribed.

Procedure:

- Construct an equilateral ΔABC.
- Inscribe a circle with center D.
- Draw the radius to each of the tangent points.
- Describe the relationships between the intersection points (E, F, G) and the elements and properties of the triangle.
- Record your data.
- Repeat the construction on other types of triangles.
- State your conjectures on the following page.



Drawings & Data

A Triangle and Its Inscribed Circle

Conjectures

Describe the relationship between the center of the circle D and the sides of the triangle; between the center D and the vertices; and between the center D and other properties of the triangle.

Explore the properties of the triangles formed by the points E, F, and G and the vertices A, B, and C.

Which conjectures for equilateral triangles are true for other types of triangles as well?

The Center of an Inscribed Circle - Part A

Task: To explore a figure formed by an inscribed circle in a triangle.

Procedure:

- Construct a ΔABC using the side-side-side option with each side 5 units long.
- Inscribe a circle in ΔABC.
- Record data.
- State your conjectures about the relationship of the center D to ΔABC.
- Repeat this procedure for other types of triangles.
- For each type of triangle, state your conjectures.

Drawings & Data

Conjectures

The Center of an Inscribed Circle - Part B

Task: To construct the center of the inscribed circle of a triangle.

Procedure:

- Construct an equilateral ΔABC.
- Find the center of the inscribed circle **without** using the "Inscribed Circle" option.
- Describe your method.
- Try this method on different types of triangles.
- State your conjectures.

Drawings & Data

Conjectures

The Center of a Circumscribed Circle - Part A

Task: To explore the relationship between the center of a circumscribed circle and the triangle which it circumscribes.

Procedure:

- Construct a ΔABC using the side-side-side option with each side 5 units long.
- Circumscribe a circle around ΔABC.
- State your conjectures about the relationship of the center D to ΔABC.
- Repeat this procedure for other types of triangles.
- For each type of triangle, state your conjectures.

Drawings & Data

Conjectures

The Center of a Circumscribed Circle - Part B

Task: To construct the center of the circumscribed circle of a triangle.

Procedure:

- Construct an equilateral ΔABC.
- Find the center of the circumscribed circle **without** using the "Circumscribed Circle" option.
- Describe your method.
- Try this method on different types of triangles.
- State your conjectures.

Drawings & Data

Conjectures

Three Consecutive Integers

Task: To explore the relationship between altitudes and the radius of the inscribed circle in triangles whose side lengths are three consecutive integers.

Procedure:

- Construct a triangle using the side-side-side option with lengths that are three consecutive integers.
- Draw the three altitudes.
- Now draw an inscribed circle in the triangle.
- Investigate the relationships between the altitudes of the triangle and the radius of the circle.
- Record your data.
- State your conjectures.

Drawings & Data

Conjectures

Reflecting Altitudes

Task: To explore the results of reflecting the altitudes of triangles.

Procedure:

- Construct an acute ΔABC.
- Draw an altitude from vertex A.
- Reflect $\overline{AC}$ in $\overline{AD}$ to form $\overline{AE}$.
- Reflect $\overline{AB}$ in $\overline{AD}$ to form $\overline{AF}$.
- Repeat this construction on an obtuse triangle.
- Predict under what conditions $\overline{AE}$ will overlap $\overline{AB}$?
- Under what conditions will $\overline{AF}$ overlap $\overline{AC}$?
- State your conjectures.

Triangle Type	Does $\overline{AE}$ overlap $\overline{AB}$?	Does $\overline{AF}$ overlap $\overline{AC}$?
Acute		
Right		
Obtuse		
Isosceles (Acute)		
Isosceles (Obtuse)		
Equilateral		

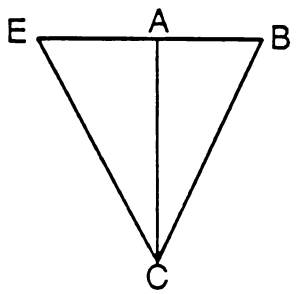
Conjectures

Reflected Images

Task: To explore the figure formed by reflecting the side(s) in one side of the triangle.

Procedure:

- Construct right ΔABC.
- Reflect $\overline{AB}$ in $\overline{AC}$, $\overline{BC}$ in $\overline{AC}$, and $\overline{AC}$ in $\overline{AC}$.
- Measure the sides and angles of ΔBAC and ΔEAC.
- We call ΔBAC the *pre-image* and ΔEAC the *image* under the reflection of the sides of ΔBAC in $\overline{AC}$.
- Record the measurements on your drawing.
- Repeat this process for other types of triangles.
- Record the measurements on your drawings.
- State conjectures that are true for all triangles and those that are true for only certain types of triangles.



Drawings & Data

Conjectures

Reflection - Part A

Task: To explore reflection in triangles.

Procedure:

- Construct an acute ΔABC.
- Reflect ΔABC in $\overline{AB}$ by reflecting $\overline{BC}$ in $\overline{AB}$, $\overline{AC}$ in $\overline{AB}$, and $\overline{AB}$ in $\overline{AB}$.
- Record the drawing.
- State your conjectures.

Drawings & Data

Conjectures

Are the two triangles congruent?

Reflection - Part B

Task: To explore reflection in triangles.

Procedure:

- Construct any ΔABC.
- Use reflections to draw another triangle that shares **only one** point with ΔABC.
- What point do the two triangles share?
- Record the drawing.
- State your conjectures.

Drawings & Data

Conjectures

Are the two triangles congruent?

Reflection - Part C

Task: To explore reflection in triangles.

Procedure:

- Construct any ΔABC.
- Use reflections to draw another triangle that shares **no** points with ΔABC.
- Record the drawing.
- State your conjectures.

Drawings & Data

Conjectures

Are the two triangles congruent?

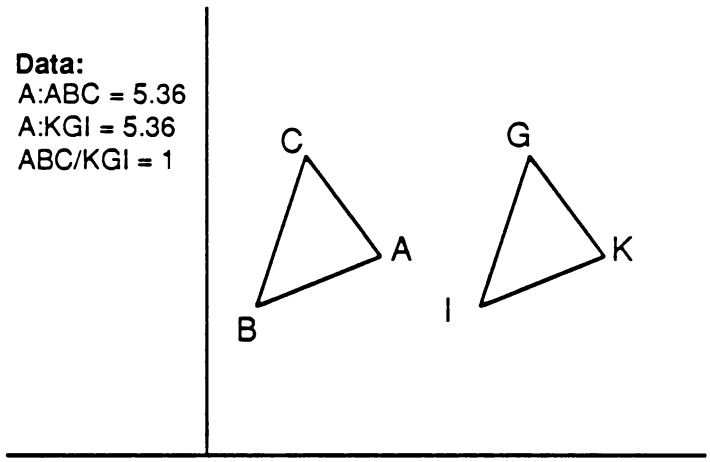
Reflection - Part D

Task: To explore reflection in triangles.

Procedure:

- Construct any ΔABC.
- Develop a method for "sliding" ΔABC to a new position such that ΔABC and its image are congruent and look like the diagram below:

Data:
A:ABC = 5.36
A:KGI = 5.36
ABC/KGI = 1



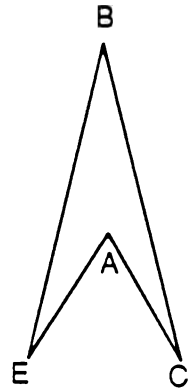
- Record the drawing.
- State your conjectures.

Drawings & Data

Conjectures

Construction Challenges

Task: To develop a procedure for reproducing this figure.



Procedure:

- Make a drawing similar to this figure.
- Collect data.
- Describe below the procedures for reproducing this figure.
- State your conjectures.

Drawings & Data

Conjectures

Procedure for reproducing figure:

For quadrilateral ACBE above, under what conditions will ∠CAE be a 90° angle?

N-Gons

Task: To use triangles and reflections to construct polygons.

Procedure:

- Using triangles and reflections, develop procedures for constructing the following shapes: square, rectangle, rhombus, kite, pentagon, octagon, nonagon, and decagon.
- On the following page, state your conjectures about constructing these polygons.

Drawings & Data

Conjectures

Square

Rectangle

Rhombus

Kite

Pentagon

Octagon

Nonagon

Decagon

Reflecting a Point to Create Triangles - Part A

Task: To explore the figure formed by reflecting the intersection point of the **altitudes** in each side of a triangle and connecting the three image points.

Procedure:

- Construct an acute ΔABC.
- Draw the three altitudes.
- Label G as their point of intersection.
- Reflect point G in each of the three sides of ΔABC producing points H, I, J.
- Draw ΔDEF and ΔHIJ.
- State your conjectures about the relationships among the points, elements, and triangles.
- Repeat the procedure for other types of triangles.

Drawings & Data

Conjectures

Reflecting a Point to Create Triangles - Part B

Task: To explore the figure formed by reflecting the intersection point of the **perpendicular bisectors** of each side of a triangle and connecting the three image points.

Procedure:

- Construct an acute ΔABC.
- Draw the three perpendicular bisectors.
- Label G as their point of intersection.
- Reflect point G in each of the three sides of ΔABC producing points H, I, J.
- Draw ΔDEF and ΔHIJ.
- State your conjectures about the relationships among the points, elements, and triangles.
- Repeat the procedure for other types of triangles.

Drawings & Data

Conjectures

Triangular Sections

Line segments divide any triangle into triangular sections. For example, an angle bisector divides a triangle into two triangular sections and three medians divide any triangle into six triangular sections.

Task: To describe methods of subdividing triangles into subtriangles that have equal area.

Procedure:

- Construct any ΔABC.
- Draw line segments.
- State your conjectures.

Drawings & Data

Conjectures

What kind of segment(s) or combinations of segments and how many of them divide(s) any triangle into **two** sections with equal areas? _____

_____

Three sections with equal areas? _____

_____

Four sections with equal areas? _____

_____

Five sections with equal areas? _____

_____

T 56
Subdividing Triangles

Task: To develop a method of subdividing triangles into congruent triangles where the number of triangles is greater than or equal to 3.

Procedure:

- Construct any ΔABC.
- Create a method.
- Describe your method.
- State your conjectures.

Drawings & Data

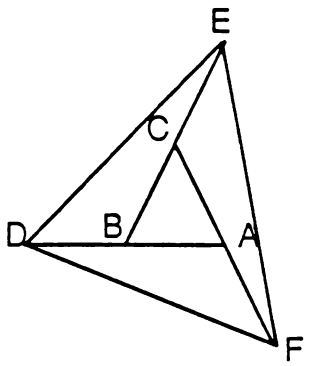
Conjectures

Extended Sides - Part A

Task: To explore relationships in figures formed by extending the sides of triangles by lengths equal to the lengths of the sides.

Procedure:

- Construct an equilateral triangle using the side-side-side option with sides that are 3 units long.
- Extend $\overline{AB}$ from B so that the extension $\overline{BD} \cong \overline{AB}$.
- Extend $\overline{BC}$ from C so that the extension $\overline{CE} \cong \overline{BC}$.
- Extend $\overline{CA}$ from A so that the extension $\overline{AF} \cong \overline{CA}$.
- Draw segments connecting the three endpoints of the extensions.
- State your conjectures.
- Repeat this procedure on two triangles with the following measurements:
  - side lengths 2–2–2
  - side lengths 4–3–3
- State your conjectures.



Drawings & Data

Conjectures

Extended Sides - Part B

Task: To explore relationships in figures formed by extending the sides of triangles by lengths equal to the lengths of the sides.

Procedure:

- Construct a triangle using the side-angle-side option so that AB = 3, $\angle BAC = 90°$, and AC = 2.
- Extend $\overline{AB}$ from B so that the extension $\overline{BD} \cong \overline{AB}$.
- Extend $\overline{BC}$ from C so that the extension $\overline{CE} \cong \overline{BC}$.
- Extend $\overline{CA}$ from A so that the extension $\overline{AF} \cong \overline{CA}$.
- Draw segments connecting the three endpoints of the extensions.
- State your conjectures.

Drawings & Data

Conjectures

Scale Change

Task: To understand the effect of the Scale change option.

Procedure:

- Draw a right ΔABC.
- Measure its sides, angles, and perimeter.
- Record your drawing of the triangle and its measurements in the space provided.
- Use the Scale change option and measure the sides, angles, and perimeter of the new ΔABC.
- Draw this scaled ΔABC below but change the labels on the vertices of the triangle.
- Replace A with A', B with B', and C with C'. ΔA'B'C' is called the image of ΔABC.
- Repeat the steps above for at least one right, one acute, one obtuse, one isosceles, and one equilateral triangle.
- Record your results.
- State your conjectures.

Drawings & Data

Right Triangle

Acute Triangle

Obtuse Triangle

Isosceles Triangle

Equilateral Triangle

Conjectures

Scaling with Medians and Altitudes

Task: To understand how a scale change affects lines drawn in a triangle and the area of the triangle.

Procedure:

- Construct an acute ΔABC.
- Draw a median and altitude from one vertex.
- Measure the sides, the median, the altitude, and the area of the triangle.
- Record the drawing of the triangle and its measurements.
- Use the Scale change option.
- Measure the sides, the median, the altitude, and the area of the new triangle.
- Record your results on the new triangle.
- Use letters with "primes" attached to denote labels for vertices (A'B'C').
- Repeat the steps above for at least one obtuse, one isosceles, and one equilateral triangle.
- State your conjectures.

Drawings & Data

Acute Triangle

Obtuse Triangle

Scaling with Medians and Altitudes

Isosceles Triangle

Equilateral Triangle

Conjectures

T 60
Triangles Inside Triangles

Task: To explore the figures formed by drawing parallels inside a triangle.

Procedure:

- Construct any ΔABC.
- Label a random point on $\overline{AB}$.
- Draw $\overline{DE}$ parallel to $\overline{BC}$ to intersect $\overline{AC}$.
- Measure the sides of ΔABC and ΔADE.
- State your conjectures and provide a convincing argument.

Drawings & Data

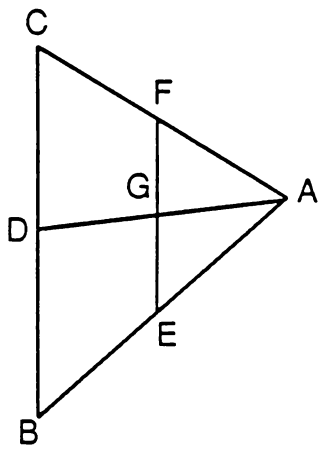
Conjectures

Midsegment and Median

Task: To explore the relationship between segments formed by a median and a midsegment.

Procedure:

- Construct any ΔABC.
- Draw median $\overline{AD}$.
- Draw midsegment $\overline{EF}$ joining sides $\overline{AB}$ and $\overline{AC}$.
- Label the intersection of $\overline{AD}$ and $\overline{EF}$ with point G.
- Measure the segments.
- State your conjectures.
- Provide supporting arguments.



Drawings & Data

Conjectures

What is the relationship between EG and FG?

Provide supporting arguments for your conjectures.

Mystery Triangles - Part A

Task: To explore a set of triangles.

Procedure:

- Construct each triangle listed in the table below and on the following page.
- Complete the tables.
- State your conjectures on the following page.

	Triangle Drawings	∠ABC	∠BCA	∠CAB
AB = 2 AC = 3 BC = 4				
AB = 3 AC = 4.5 BC = 6				
AB = 4.5 AC = 6.75 BC = 9				

Mystery Triangles - Part A

	Triangle Drawings	∠ABC	∠BCA	∠CAB
AB = 10 AC = 9 BC = 8				
AB = 7.5 AC = 6.75 BC = 6				
AB = 5 AC = 4.5 BC = 4				

Conjectures

Mystery Triangles - Part B

Task: To explore a set of triangles.

Procedure:

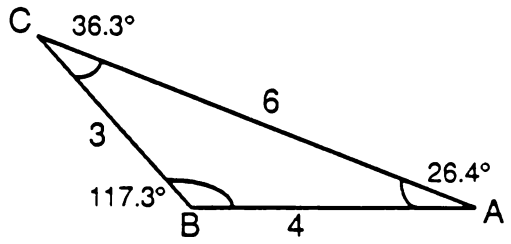
- Construct each triangle listed in the table below and on the following page.
- Complete the tables.
- State your conjectures on the following page.

	Triangle Drawings	AB	BC	AC	∠ACB
∠BAC = 30 AB = 4 ∠CBA = 70					
∠BAC = 30 AB = 5 ∠CBA = 70					
∠BAC = 30 AB = 6 ∠CBA = 70					

Conjectures

Constructing Similar Triangles

Task: To describe different methods for constructing triangles which are similar, in the geometric sense, to the triangle given in the drawing:



Procedure:

- Make a drawing similar to the figure above.
- Collect data.
- Describe below the procedure for constructing a similar figure.
- State your conjectures.

Drawings & Data

Conjectures

Describe your method for constructing a similar figure:

Dividing Sides - Part A

Task: To explore relationships between a triangle and a smaller triangle formed using subdivision.

Procedure:

- Construct any ΔABC.
- Subdivide $\overline{AC}$ and $\overline{AB}$ into three equal parts.
- Draw $\overline{DF}$.
- Measure the elements of the triangles.
- Repeat the drawing on different triangles.
- State your conjectures about the relationship between ΔABC and ΔADF.

Drawings & Data

Conjectures

Dividing Sides - Part B

Task: To explore relationships between a triangle and a smaller triangle formed using subdivision.

Procedure:

- Use your construction from Part A:
  - Construct any ΔABC.
  - Subdivide $\overline{AC}$ and $\overline{AB}$ into three equal parts.
  - Draw $\overline{DF}$.
- Draw a median from vertex A in ΔABC.
- Draw a median from vertex A in ΔADF.
- Measure the elements of the triangles.
- Repeat the drawing on three other triangles.
- State your conjectures about the relationship between ΔABC and ΔADF.

Drawings & Data

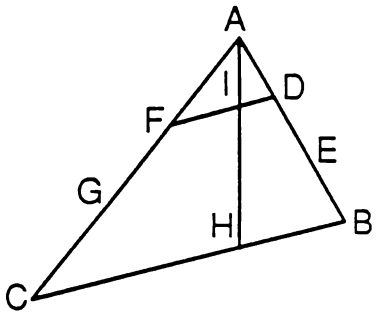
Conjectures

Dividing Sides - Part C

Task: To explore relationships between a triangle and a smaller triangle formed using subdivision.

Procedure:

- Use your construction from Part B:
  - Construct any ΔABC.
  - Subdivide $\overline{AC}$ and $\overline{AB}$ into three equal parts.
  - Draw $\overline{DF}$.
- Label a random point H on $\overline{BC}$.
- Draw $\overline{AH}$ and label point I as the intersection of $\overline{FD}$ and $\overline{AH}$.
- Measure $\overline{BH}$, $\overline{CH}$, $\overline{DI}$, and $\overline{FI}$.
- Find relationships.
- Label another random point on $\overline{BC}$ and repeat the procedure.
- State your conjectures.



Drawings & Data

Conjectures

T 65
Creating Similar Triangles

A line segment inside a triangle can divide a triangle into two similar triangles. In some cases these two triangles are also similar to the original triangle; in other cases, the line segment only creates one triangle which is similar to the original triangle.

Task: To explore conditions under which a line segment divides a triangle into similar triangles.

Procedure:

- Inside any ΔABC draw any line segment using options on the Draw and Label menus.
- Determine if and how many similar triangles are created.
- Record information in the chart below.
- Repeat the construction for different types of triangles and record your findings.
- Repeat the procedure for different types of line segments.
- State your conjectures on the following page.

Description of Segment Drawn	Type of Triangle	One similar triangle	Two similar triangles	Three similar triangles

Conjectures

Similar Triangles That Don't Touch - Part A

Task: To describe methods for drawing a triangle inside ΔABC that is similar to ΔABC such that the two triangles share no points.

Procedure:

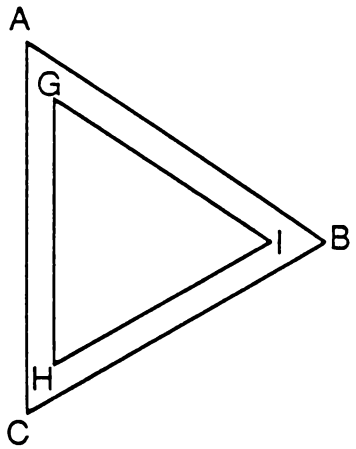
- Draw any ΔABC.
- Draw the similar triangle so that it is located anywhere inside ΔABC.
- Provide data to verify that your methods work.
- State your conjectures.

Drawings & Data

Conjectures

Similar Triangles That Don't Touch - Part B

Task: To describe methods for drawing a triangle inside ΔABC that is similar to ΔABC such that the sides are equidistant from the corresponding sides of ΔABC.



Procedure:

- Draw any ΔABC.
- Draw a similar triangle inside ΔABC such that the sides are **equidistant** from the corresponding sides of ΔABC.
- Provide data to verify that your methods work.
- State your conjectures.

Drawings & Data

Conjectures

Trigonometric Ratios

The data in the chart below represents the measurements of one side (AB = 5) and three angles in a set of right triangles where ∠A = 90°. The trigonometric functions sine, tangent, and cosine are defined for ∠B.

Task: To explore trigonometric functions in right triangles.

Procedure:

- Construct each right triangle listed below.
- Compute the ratios CA/CB, CA/AB, and AB/CB.
- Record the ratios.
- State your conjectures.

∠A	∠B	∠C	AB	sin ∠B (CA/CB)	tan ∠B (CA/AB)	cos ∠B (AB/CB)
90°	10°	80°	5	.17	.18	.98
90°	15°	75°	5	.26	.27	.97
90°	20°	70°	5			
90°	25°	65°	5			
90°	30°	60°	5			
90°	40°	50°	5			
90°	45°	45°	5			
90°	50°	40°	5			
90°	60°	30°	5			
90°	70°	20°	7			
90°	80°	10°	7			
90°	85°	5°	4			

Conjectures

State conjectures about the behavior of the ratios (the values of the trigonometric functions) as ∠B and ∠C either increase or decrease in size.

_____

_____

_____

Constant Perimeter - Part A

The chart below lists the lengths of the sides of four triangles. Each triangle has a perimeter of 21 units.

Task: To explore triangles that have the same perimeter.

Procedure:

- Draw each triangle using the side-side-side option and the measurements in the chart below.
- Use the Repeat option "on previous shape" to scan the four triangles.
- Record any changes that you observe as you scan the triangles.
- Choose one of the triangles.
- Draw its inscribed circle.
- Draw a radius to one of the tangent points.
- Using the Repeat option, scan the four triangles with their inscribed circle and radii.
- Record any changes that you observe.
- State your conjectures.

AB	AC	BC	Triangles
7	7	7	
7	4	10	
8	4	9	
6	7	8	

Conjectures

Constant Perimeter - Part B

Task: To explore triangles that have the same perimeter.

Procedure:

- Using the same triangles from Part A, measure the perimeter, and the area of the triangle, and the radius of the inscribed circle for each triangle.
- Fill in the table below.
- State your conjectures.

AB	AC	BC	Area	Perimeter	Radius of inscribed circle
7	7	7			
7	4	10			
8	4	9			
6	7	8			

Conjectures

Constant Area

The table below contains four triangles whose areas are 10 square units.

Task: To explore triangles that have the same area.

Procedure:

- Draw each triangle using the side-angle-side option and the measurements in the chart below.
- Find the perimeter of each triangle and the length of the radius of the inscribed circle.
- State your conjectures.

Side	Angle	Side	**Drawing**	Area	Perimeter	Radius
5	90°	4.00				
6	90°	3.33				
7	90°	2.86				
10	90°	2.00				

Conjectures

T 70
Center of the Circumscribed Circle

Task: To investigate relationships in a figure formed by triangles that are circumscribed by circles.

Procedure:

- Construct any ΔABC.
- Circumscribe ΔABC with a circle having center D.
- Repeat the construction on different types of triangles.
- Record the diagrams.
- Make measurements.
- State your conjectures.

Drawings & Data

Conjectures

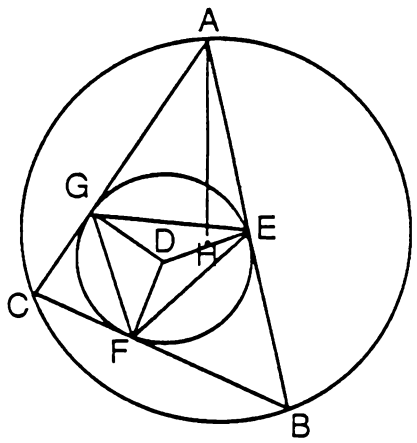
The location of the center D relative to the triangle:

The relative size of the circle and the triangle:

The relationship of point D to ΔABC:

T 71
Two Circles

Task: To investigate figures formed by a triangle and both the inscribed and the circumscribed circles of the triangle.



Procedure:

- Construct an acute ΔABC.
- Inscribe a circle with center D.
- Draw three radii to the tangent points E, F, G.
- Draw ΔEFG.
- Circumscribe a circle with center H about ΔABC.
- Draw a radius $\overline{HA}$.
- Make measurements.
- Repeat the procedure on other types of triangles.
- State your conjectures.

Drawings & Data

Conjectures

State conjectures about relationships between a) the two circles, b) the two triangles, and c) the circles and the triangles. In particular, look at different types of triangles for a relationship between the radii of the two circles. Identify in which type of triangle the areas of the two circles are closest.

Midpoints and Circumscribed Circles - Part A

Task: To investigate the relationship between the circumscribed circle of a triangle and the circle which circumscribes the smaller triangle formed by connecting the midpoints of the sides of the original triangle.

Procedure:

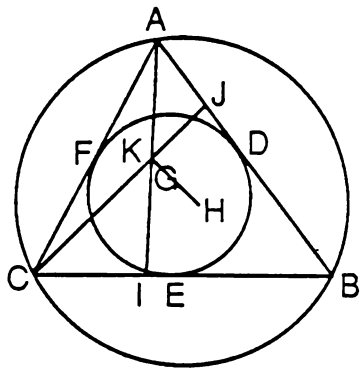
- Construct any acute ΔABC.
- Bisect the three sides producing points D, E, and F.
- Draw a circumscribed circle around ΔDEF and ΔABC.
- Make measurements.
- Repeat the procedure for other triangles and collect data.
- State your conjectures about the relationships between the two circles.

Drawings & Data

Conjectures

Midpoints and Circumscribed Circles - Part B

Task: To investigate the relationship between the circumscribed circle of a triangle and the circle which circumscribes the smaller triangle formed by connecting the midpoints of the sides of the original triangle.



Procedure:

- Use your construction from Part A:
  - Construct an acute ΔABC.
  - Bisect the three sides producing points D, E, and F.
  - Draw a circumscribed circle with center G around ΔDEF and one with center H around ΔABC.
- Draw any two altitudes in ΔABC and label their intersection with point K.
- Make measurements.
- Repeat the procedure for other triangles and collect data.
- State your conjectures about the relationship among points G, H, and K.

Drawings & Data

Conjectures

What is the relationship between points:

G - The center of the circumcircle of ΔDEF. _____

H - The center of the circumcircle of ΔABC. _____

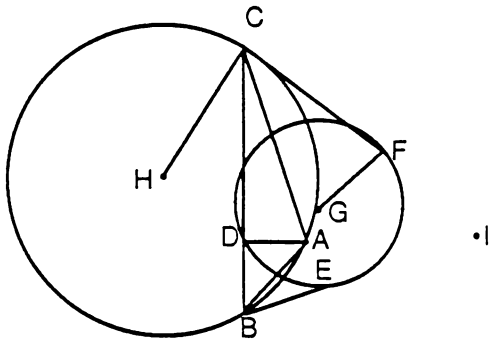
K - The intersection point of the altitudes. _____

_____

_____

Altitudes and Circumscribed Circles

Task: To investigate the relationships between the center of the circumscribed circle of a triangle and the center of the circle which intersects the feet of the altitudes of a triangle. To investigate the relationship between the radii of these circles.



Procedure:

- Construct an obtuse ΔABC.
- Draw the three altitudes $\overline{AD}$, $\overline{BE}$, $\overline{CF}$.
- Circumscribe a circle with center G about ΔDEF.
- Circumscribe a circle with center H about ΔABC.
- Draw radius $\overline{HC}$ and radius $\overline{GF}$.
- Label the intersection point I of the three altitudes.
- Repeat the procedure with other triangles and investigate relationships among the points in the drawings.
- State your conjectures.

Drawings & Data

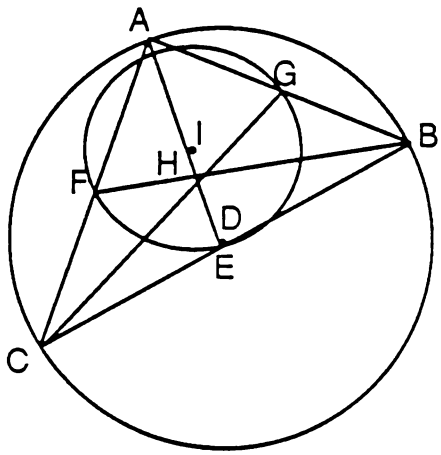
Conjectures

Medians and Circumscribed Circles

Task: To investigate the relationship between the centers of the circumscribed circle of a triangle and the circle which intersects the feet of the medians of a triangle.

Procedure:

- Construct ΔABC.
- Draw the circumscribed circle D about ΔABC.
- Draw the three medians: $\overline{AE}$, $\overline{BF}$, and $\overline{CG}$.
- Label their intersection point H.
- Draw the circumscribed circle I about ΔEFG.
- Investigate relationships between the three points D, H, and I.
- Repeat this procedure on other types of triangles.
- State your conjectures.



Drawings & Data

Conjectures

Information on The Geometric Supposer Society

The Geometric Supposer Society is an informal group of educators who use *The Geometric Supposer* in their classrooms. Its purpose is to provide a forum for the exchange of ideas for using *The Supposer* in teaching geometry.

Membership is open to educators who send in a detailed description of how they use *The Supposer* in their classes.

Sunburst compiles the contributed ideas into a booklet and sends it to all the members.

Send a detailed description of your *Supposer* classroom application, including the grade level, the specific course (regular, accelerated, etc.), the content involved, and the activity. Include a copy of any student problem sheets you use. Let us know how your students responded. Share their surprising conjectures.

Send your ideas to:

The Geometric Supposer Society
Sunburst Communications, Inc.
39 Washington Avenue
Pleasantville, NY 10570

If you have questions:

In the United States call toll-free (800) 431-1934
In Canada call toll-free (800) 247-6756
Or call collect (914) 769-5030